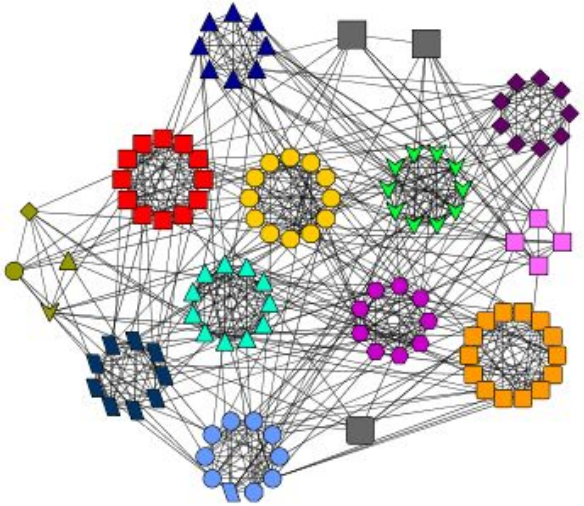
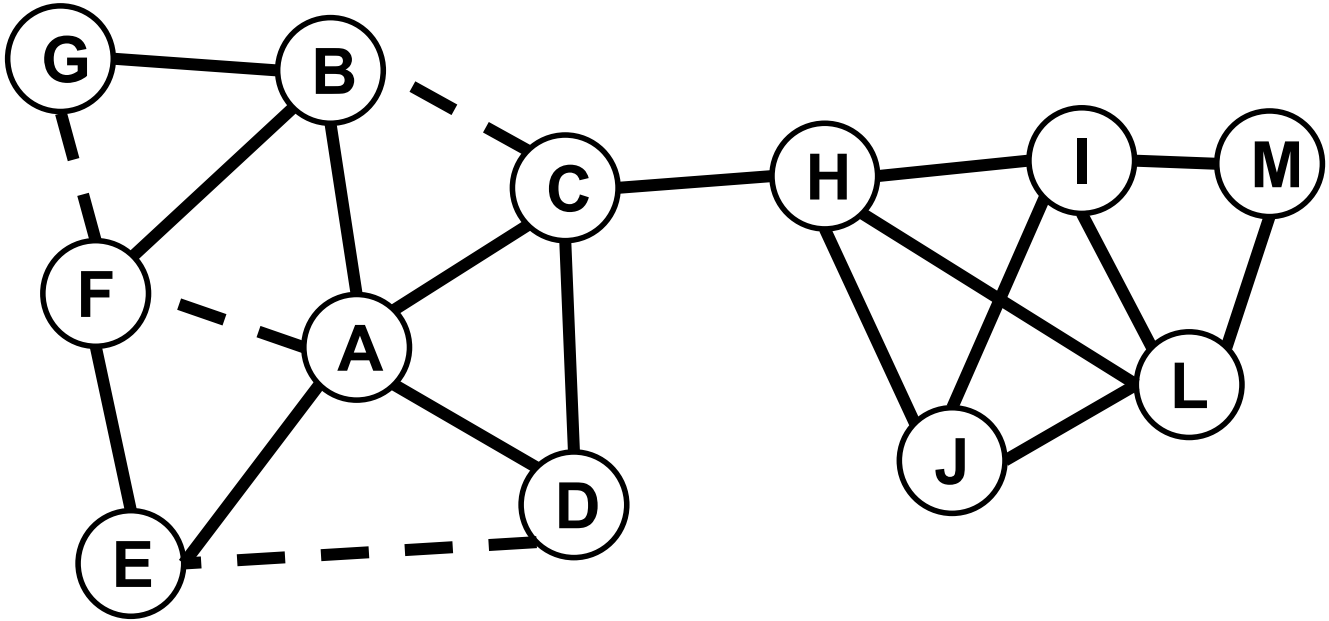


Universidade Federal do ABC

Comunidades

Prof. Fabrício Olivetti de França

Comunidades

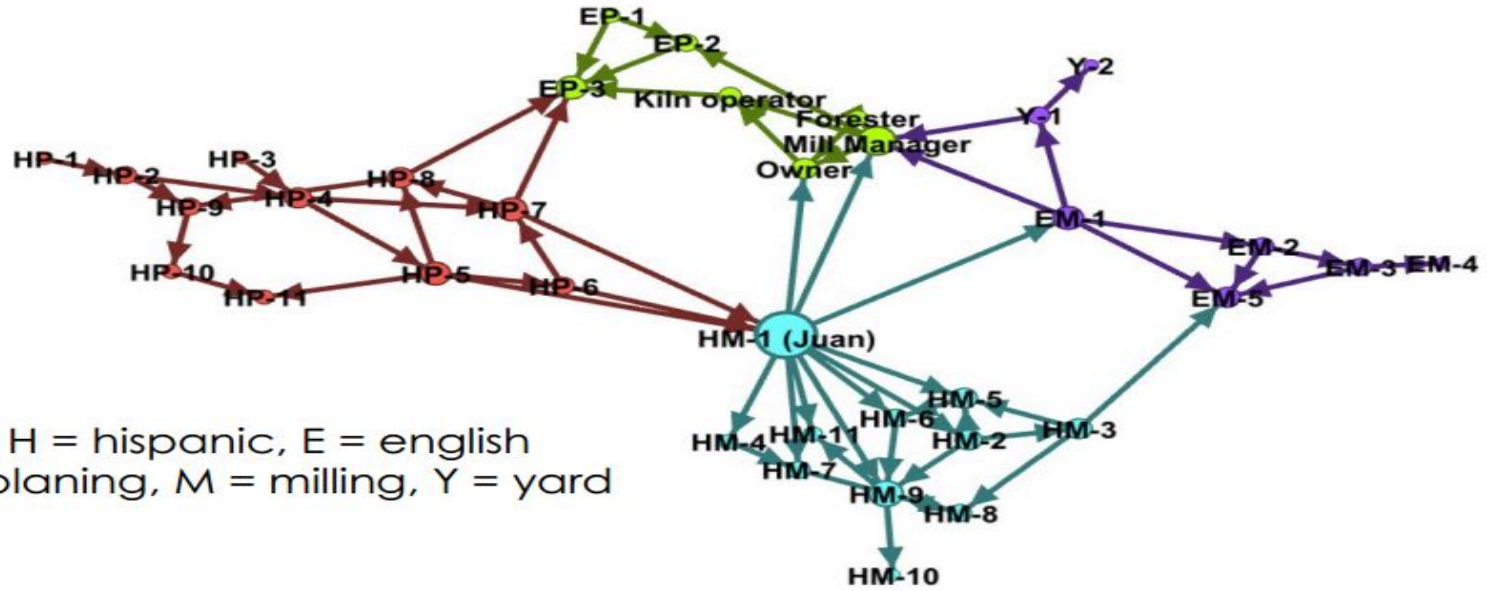


Particionamento de Redes

- Redes sociais: comunidades semelhantes
- Redes biológicas: funcionalidades de genes

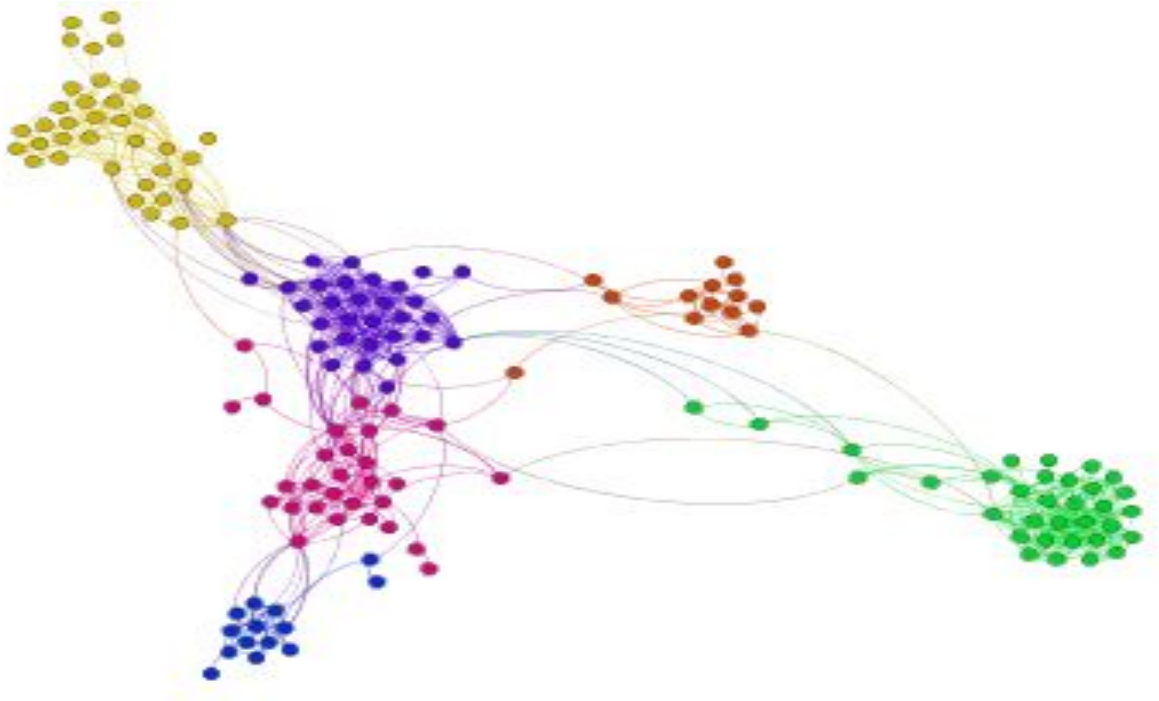
Estudar como funciona o processo de comunicação e difusão de informação na rede.

Dinâmica da Comunicação



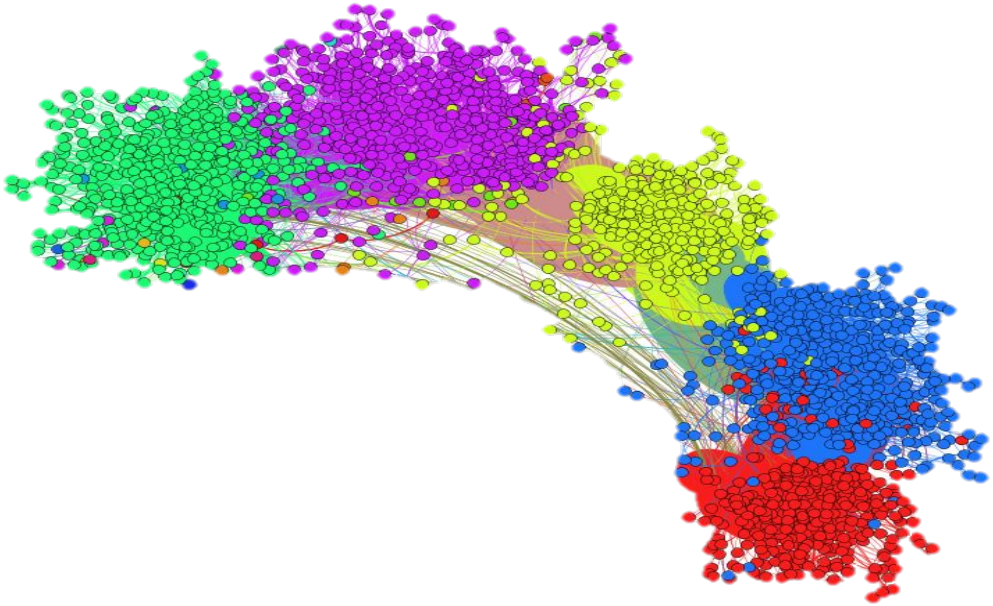
Sawmill network: source Exploratory Social Network Analysis with Pajek

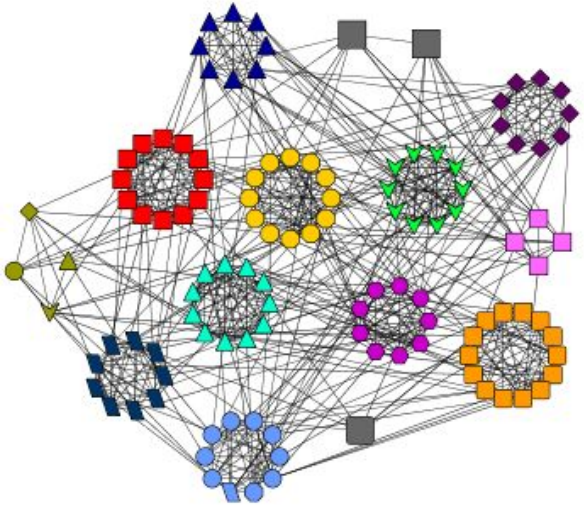
Particionamento Visual



Universidade Federal do ABC

Particionamento Visual





Universidade Federal do ABC

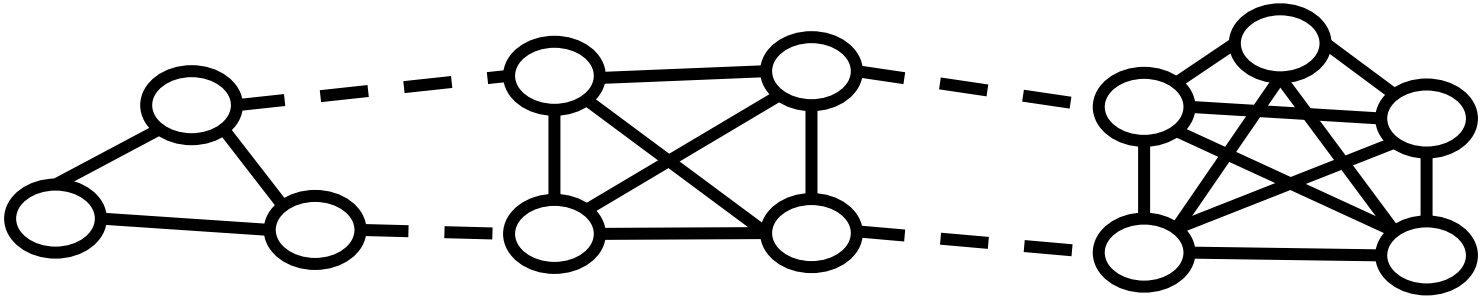
Métodos de Particionamento

Prof. Fabrício Olivetti de França

Métodos de Particionamento

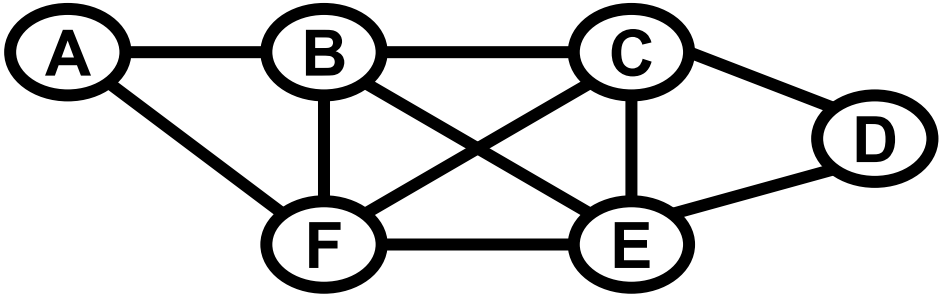
- **Ligações mútuas:** todos conhecem todos em um grupo
- **Ligações frequentes:** todos conhecem a maioria do grupo
- **Proximidade:** todos estão separados por no máximo n nós

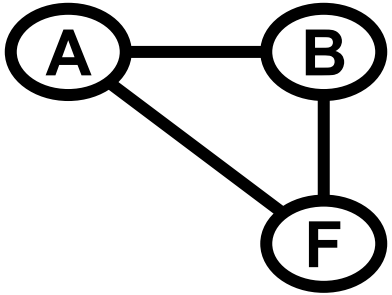
Particionamento por Cliques

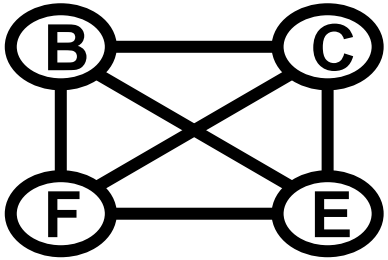


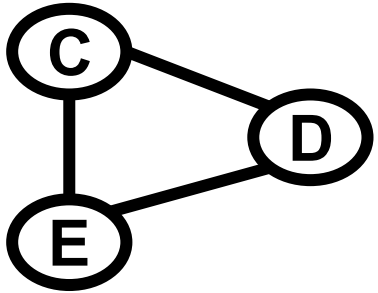
Clique de um Grafo

CLIQUE



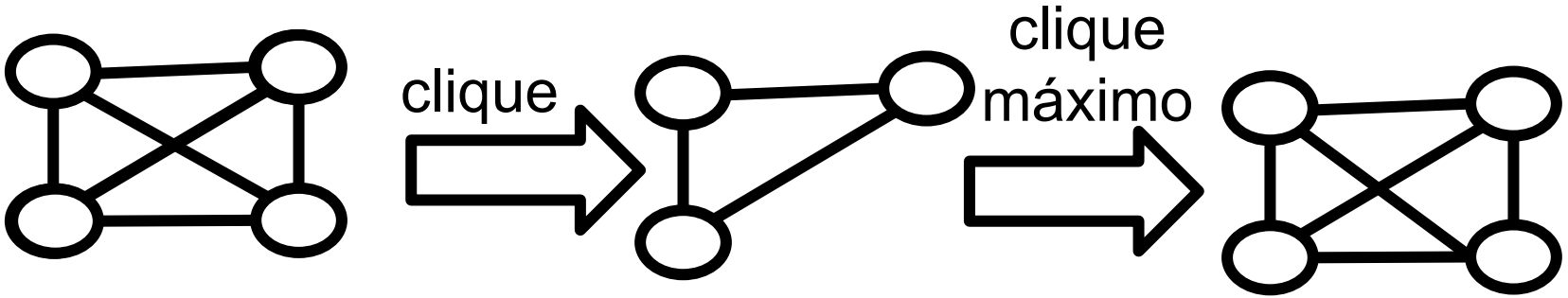






Universidade Federal do ABC

Clique de um Grafo

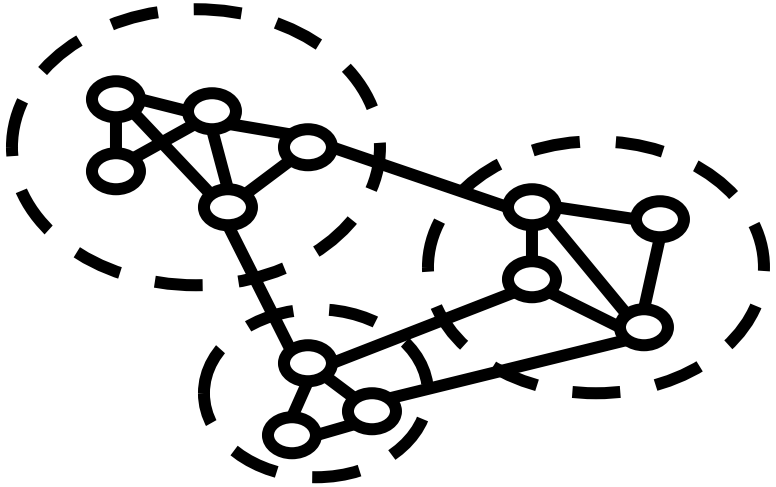


Clique de um Grafo

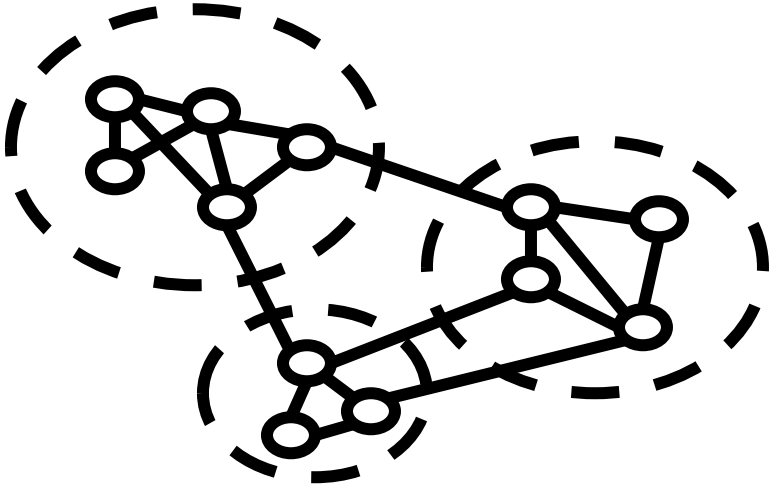
$$\frac{n(n-1)\dots(n-k+1)}{1\cdot 2\cdot \ldots \cdot k} = \frac{n!}{k!(n-k)!} = \binom{n}{k}$$

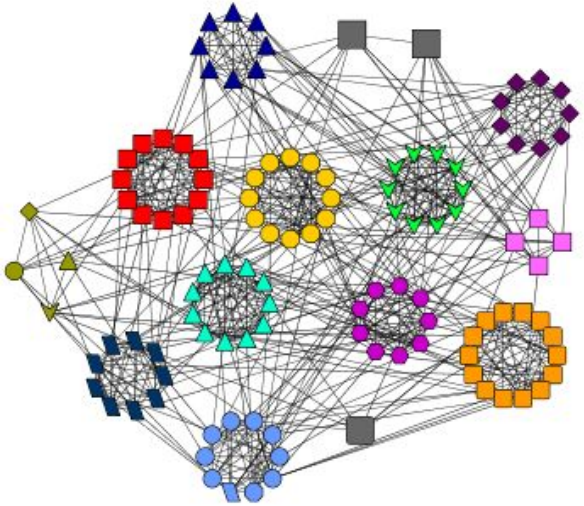
$$\sum_{1\leq k\leq n} \binom{n}{k} = 2^n - 1$$

Particionamento por Cliques



k-CORE



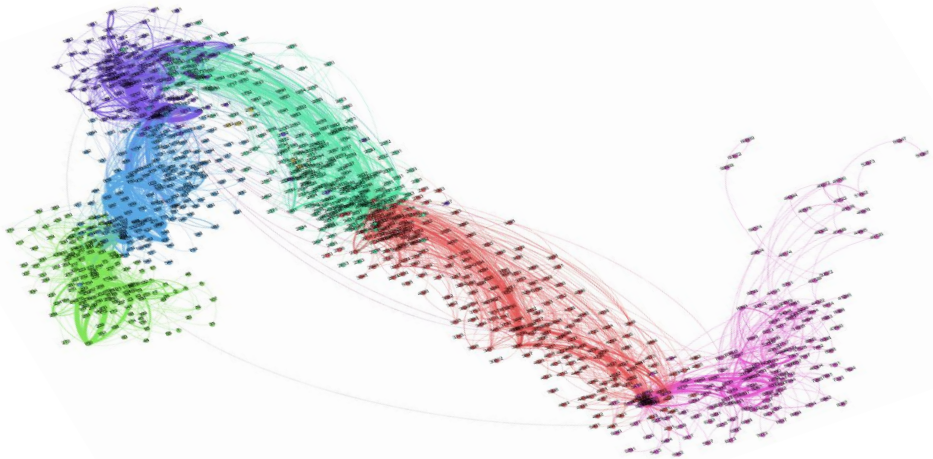


Universidade Federal do ABC

Girvan-Newman

Prof. Fabrício Olivetti de França

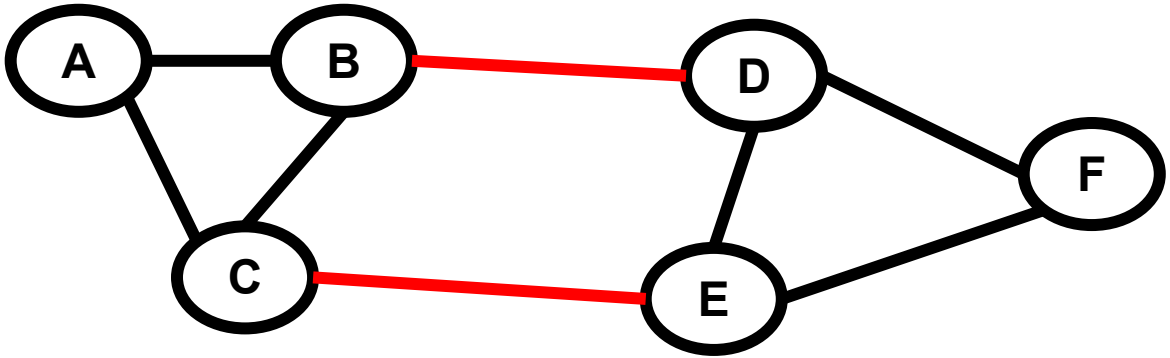
Centralidade



Métodos Girvan-Newman

Centralidades:

(A,B) = 0,167
(A,C) = 0,167
(B,C) = 0,133
(B,D) = **0,300**
(C,E) = **0,300**
(D,E) = 0,133
(D,F) = 0,167
(E,F) = 0,167

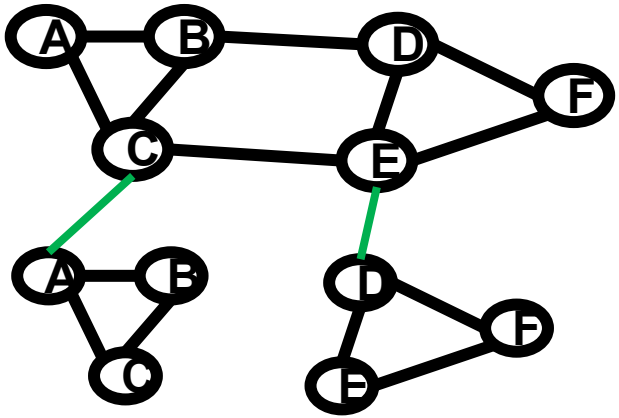


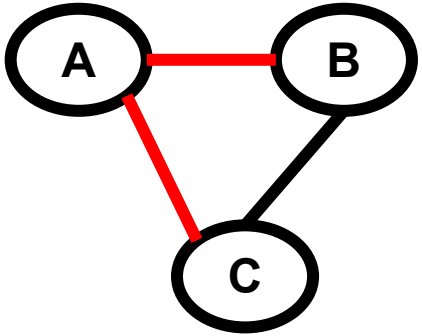
Universidade Federal do ABC

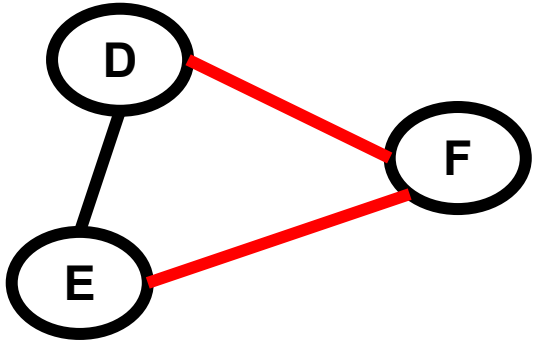
Métodos Girvan-Newman

Centralidades:

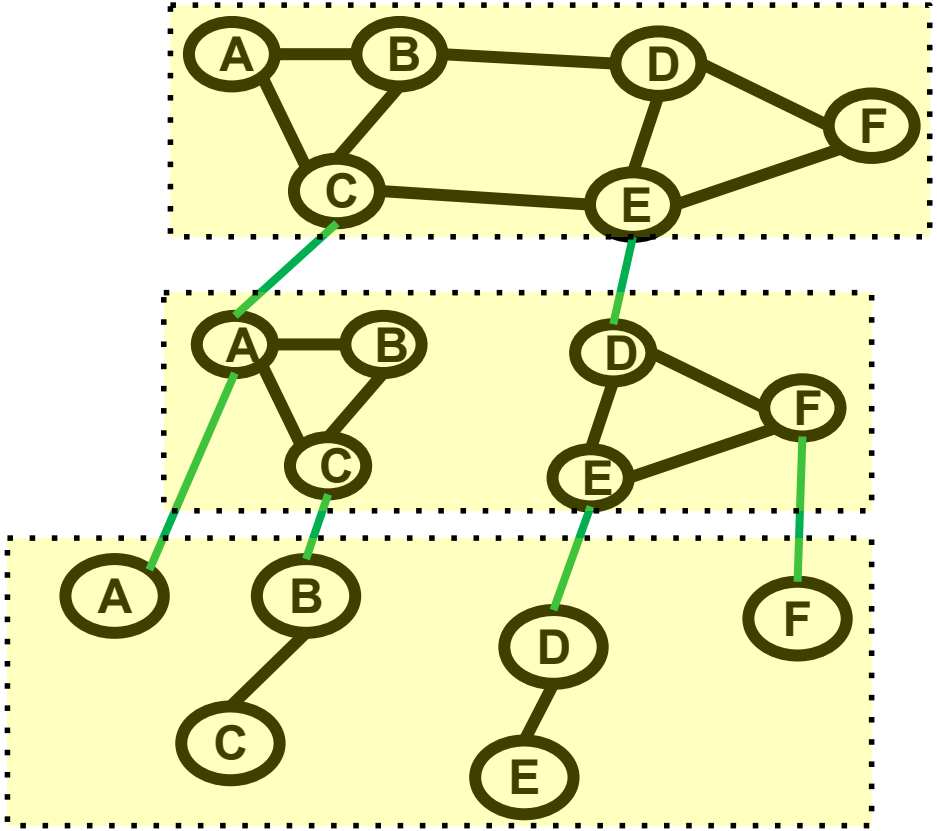
(A,B) = **0,167**
(A,C) = **0,167**
(B,C) = 0,133
(B,D) = 0,000
(C,E) = 0,000
(D,E) = 0,133
(D,F) = **0,167**
(E,F) = **0,167**







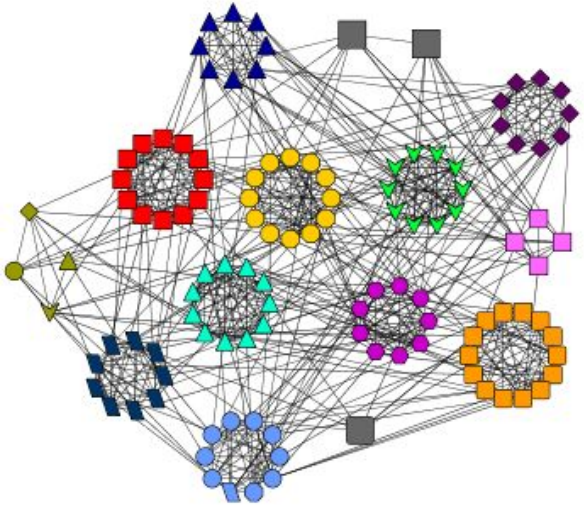
Métodos Girvan-Newman



Universidade Federal do ABC

Métodos Girvan-Newman no Facebook

- http://www.touchgraph.net/
- http://www1.maths.leeds.ac.uk/statistics/workshop/lasr2006/proceedings/pinney-talk.pdf



Universidade Federal do ABC

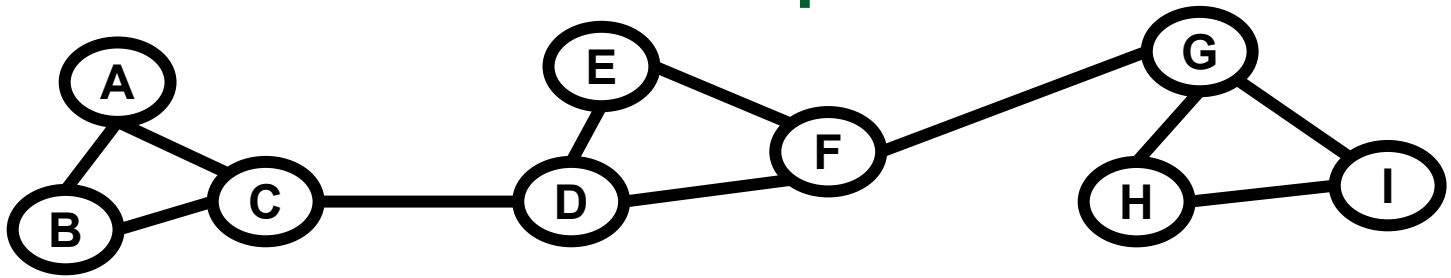
Particionamento Espectral

Prof. Fabrício Olivetti de França

Particionamento Espectral

$$L = G - A$$

Particionamento Espectral



	A	B	C	D	E	F	G	H	I
A	2	-1	-1	0	0	0	0	0	0
B	-1	2	-1	0	0	0	0	0	0
C	-1	-1	3	-1	0	0	0	0	0
D	0	0	-1	3	-1	-1	0	0	0
E	0	0	0	-1	2	-1	0	0	0
F	0	0	0	-1	-1	3	-1	0	0
G	0	0	0	0	0	-1	3	-1	-1
H	0	0	0	0	0	0	-1	2	-1
I	0	0	0	0	0	0	-1	-1	2

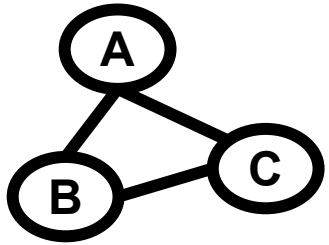
Matriz Laplaciana

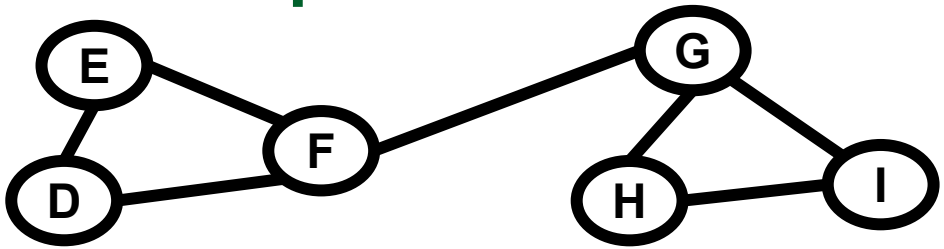
Autovalores

0,0	0,2	0,7	3,0	3,0	3,0	3,0	4,3	4,8

Universidade Federal do ABC

Particionamento Espectral





	A	B	C	D	E	F	G	H	I
A	2	-1	-1	0	0	0	0	0	0
B	-1	2	-1	0	0	0	0	0	0
C	-1	-1	2	0	0	0	0	0	0
D	0	0	0	2	-1	-1	0	0	0
E	0	0	0	-1	2	-1	0	0	0
F	0	0	0	-1	-1	3	-1	0	0
G	0	0	0	0	0	-1	3	-1	-1
H	0	0	0	0	0	0	-1	2	-1
I	0	0	0	0	0	0	-1	-1	2

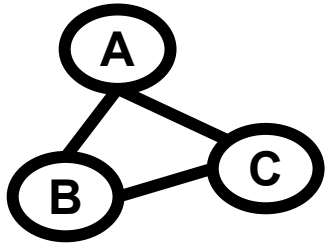
Matriz Laplaciana

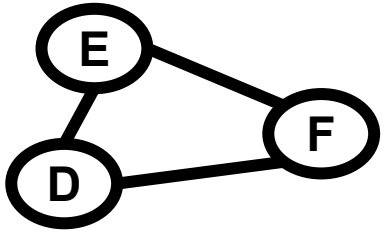
Autovalores

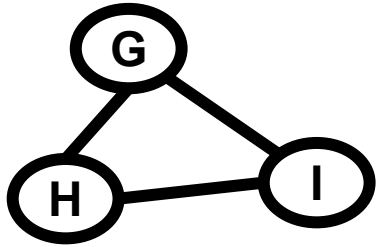
0,0	0,0	0,4	3,0	3,0	3,0	3,0	3,0	4,6

Universidade Federal do ABC

Particionamento Espectral







	A	B	C	D	E	F	G	H	I
A	2	-1	-1	0	0	0	0	0	0
B	-1	2	-1	0	0	0	0	0	0
C	-1	-1	2	0	0	0	0	0	0
D	0	0	0	2	-1	-1	0	0	0
E	0	0	0	-1	2	-1	0	0	0
F	0	0	0	-1	-1	2	0	0	0
G	0	0	0	0	0	0	2	-1	-1
H	0	0	0	0	0	0	-1	2	-1
I	0	0	0	0	0	0	-1	-1	2

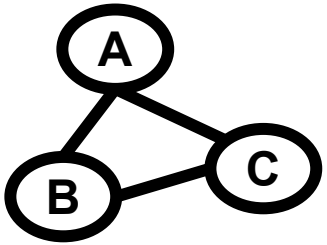
Matriz Laplaciana

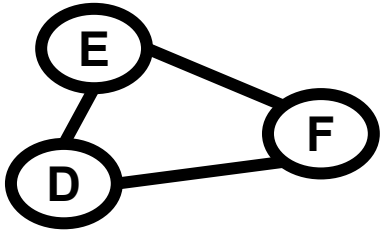
Autovalores

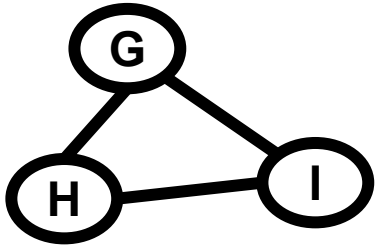
0,0	0,0	0,0	3,0	3,0	3,0	3,0	3,0	3,0

Universidade Federal do ABC

Particionamento Espectral







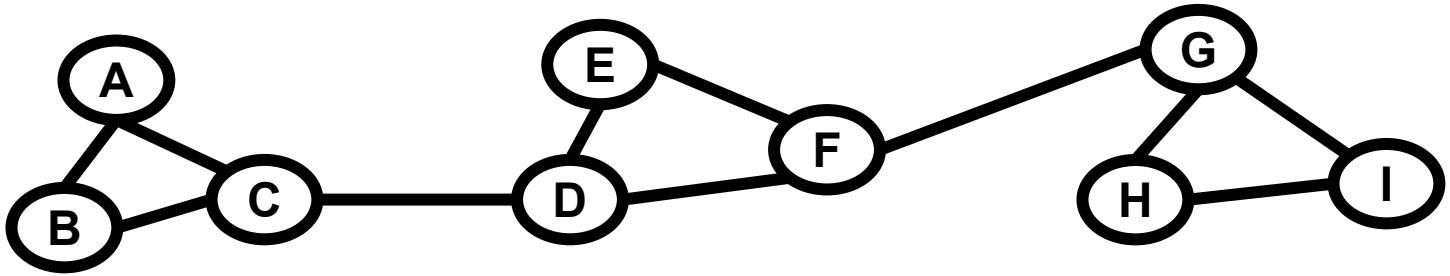
Autovalores

0,0	0,0	0,0	3,0	3,0	3,0	3,0	3,0	3,0

Autovetores

0,6	0,6	0,6	0,0	0,0	0,0	0,0	0,0	0,0
0,0	0,0	0,0	0,6	0,6	0,6	0,0	0,0	0,0
0,0	0,0	0,0	0,0	0,0	0,0	0,6	0,6	0,6

Particionamento Espectral



Autovalores

0,0	0,2	0,7	3,0	3,0	3,0	3,0	4,3	4,8

Autovetores

0,4	0,4	0,3	0,1	0,0	-0,1	-0,3	-0,4	-0,4
0,3	0,3	0,1	-0,4	-0,6	-0,4	0,1	0,3	0,3

Universidade Federal do ABC

Particionamento Espectral

- Tem autovalor correspondente maior que **ZERO** se e somente se a rede for conectada e;

- Particiona a rede em duas redes distintas repartindo em torno de um nó que divide a rede de forma simétrica.

Particionamento Espectral

$$\frac{4}{nD} \leq \sigma \leq 1$$

Particionamento Espectral

- autovalor > 0 = Grupo 1
- autovalor < 0 = Grupo 2

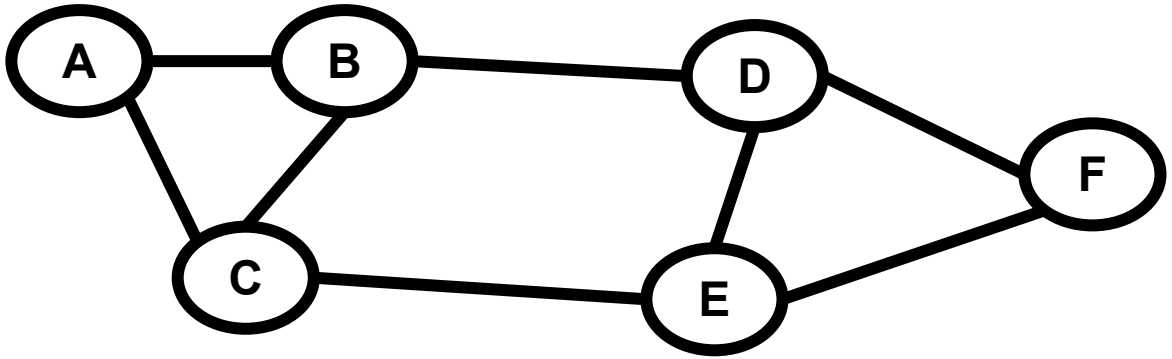
Particionamento Espectral

G

2					
	3				
		3			
			3		
				3	
					2

A

0	1	1	0	0	0
1	0	1	1	0	0
1	1	0	0	1	0
0	1	0	0	1	1
0	0	1	1	0	1
0	0	0	1	1	0



Universidade Federal do ABC

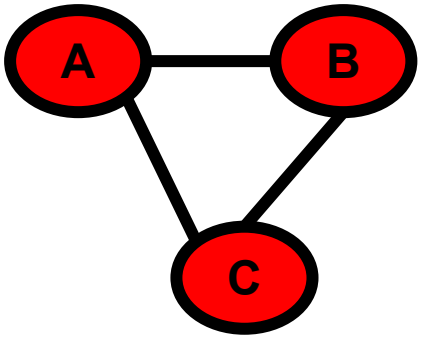
Particionamento Espectral

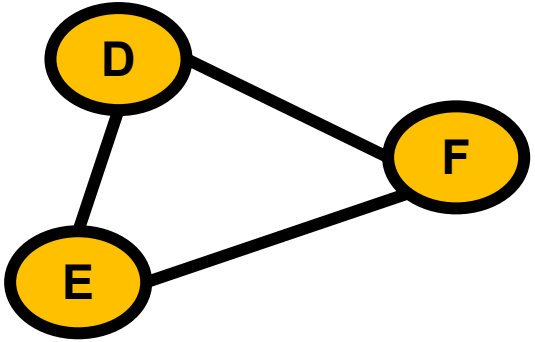
L

2	-1	-1	0	0	0
-1	3	-1	-1	0	0
-1	-1	3	0	-1	0
0	-1	0	3	-1	-1
0	0	-1	-1	3	-1
0	0	0	-1	-1	2

Autovetor Fiedler

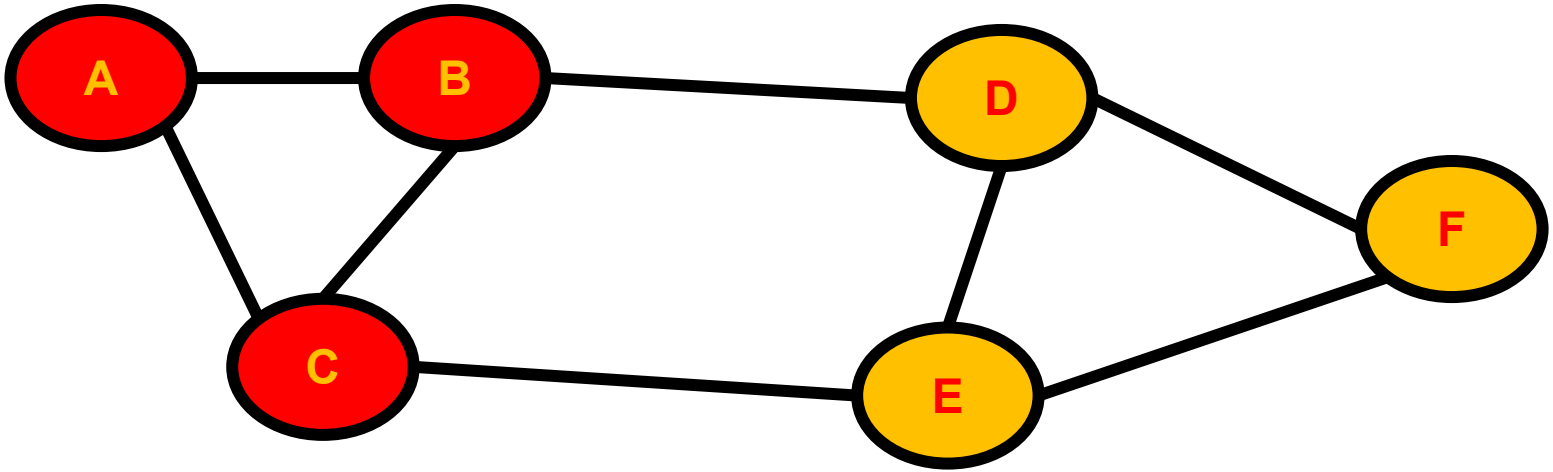
0,5773	Grupo 1
0,2887	Grupo 1
0,2887	Grupo 1
-0,2887	Grupo 2
-0,2887	Grupo 2
-0,5773	Grupo 2





Universidade Federal do ABC

Particionamento Espectral



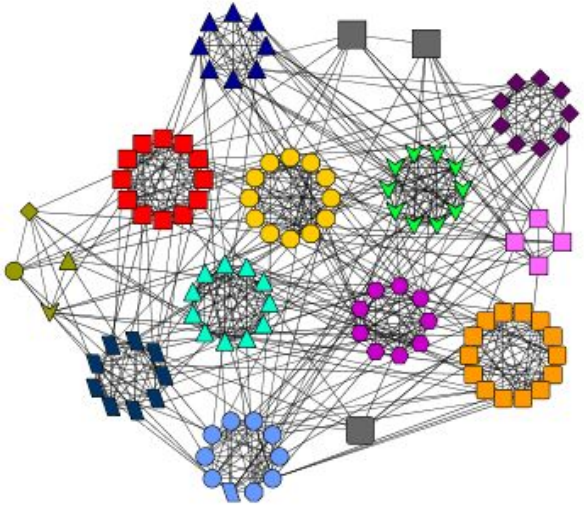
Particionamento Espectral

A. Pothen, H. Simon, K.-P. Liou, “Partitioning sparse matrices with eigenvectors of graphs”, SIAM J. Mat. Anal. Appl. 11:430-452 (1990)

M. Fiedler, “Algebraic Connectivity of Graphs”, Czech. Math. J., 23:298-305 (1973)

M. Fiedler, Czech. Math. J., 25:619-637 (1975)

Universidade Federal do ABC

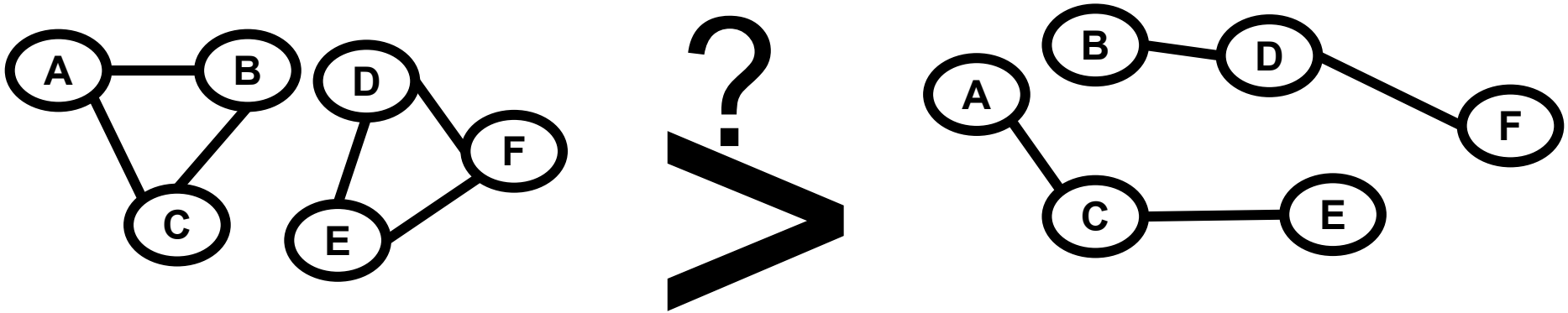


Universidade Federal do ABC

Modularidade

Prof. Fabrício Olivetti de França

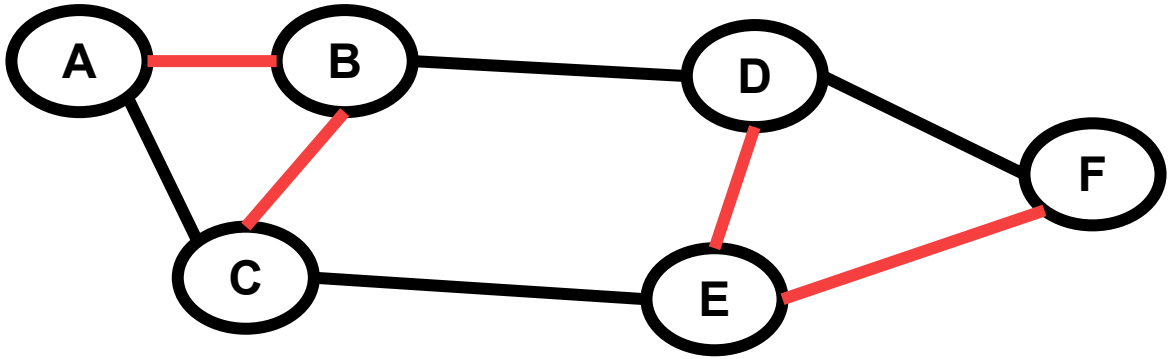
Modularidade



Modularidade

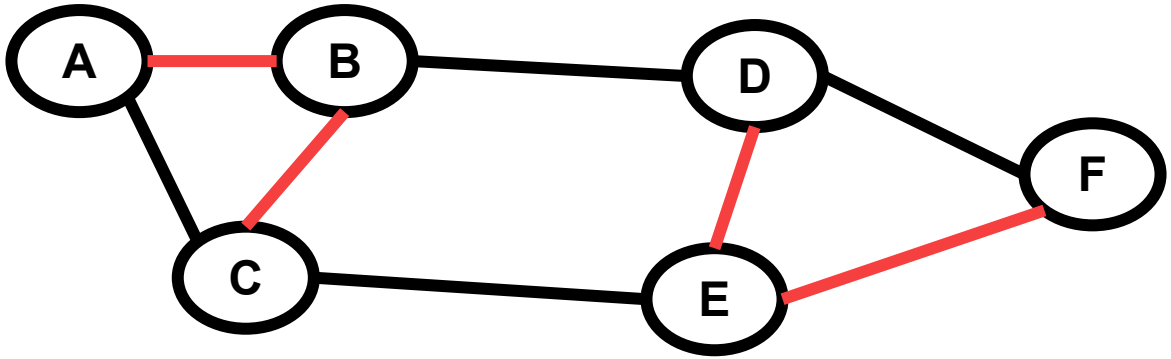
$$Q = \sum_i \left(e_{i,i} - \left(\sum_j e_{i,j} \right)^2 \right)$$

Maximizar modularidade



Maximizar modularidade

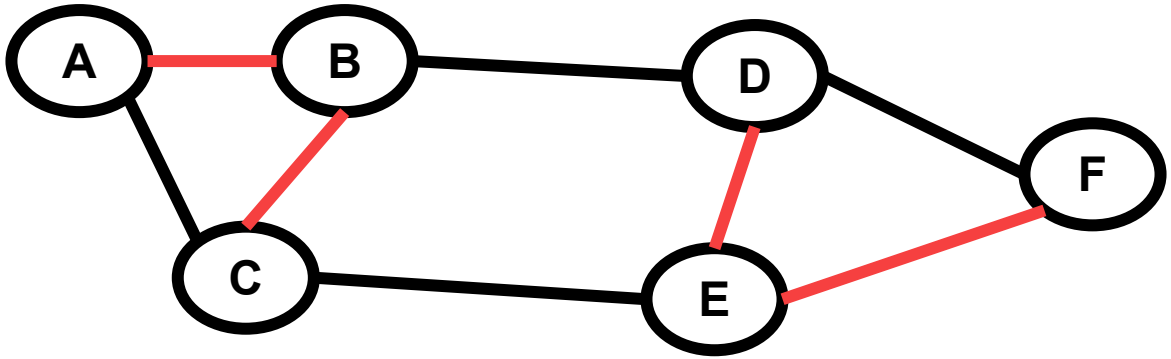
e11 = e22 = 2/8



Universidade Federal do ABC

Maximizar modularidade

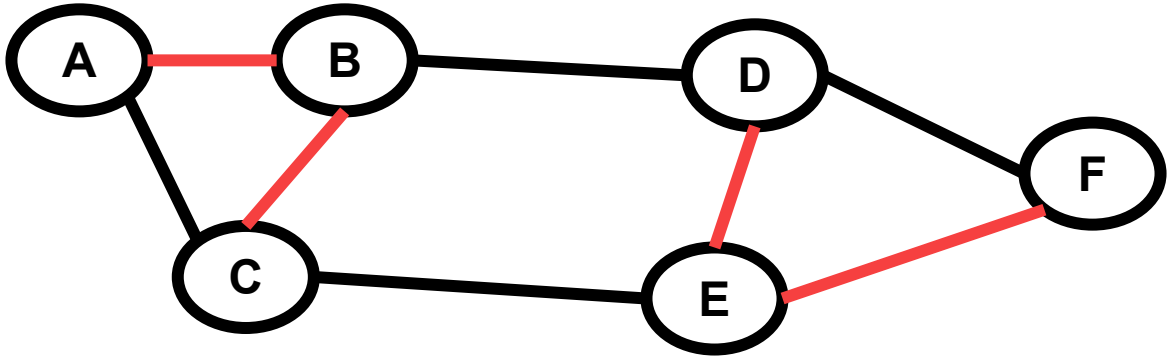
e12 = e21 = 2/8



Maximizar modularidade

$$E = \begin{bmatrix} 2/8 & 2/8 \\ 2/8 & 2/8 \end{bmatrix}$$

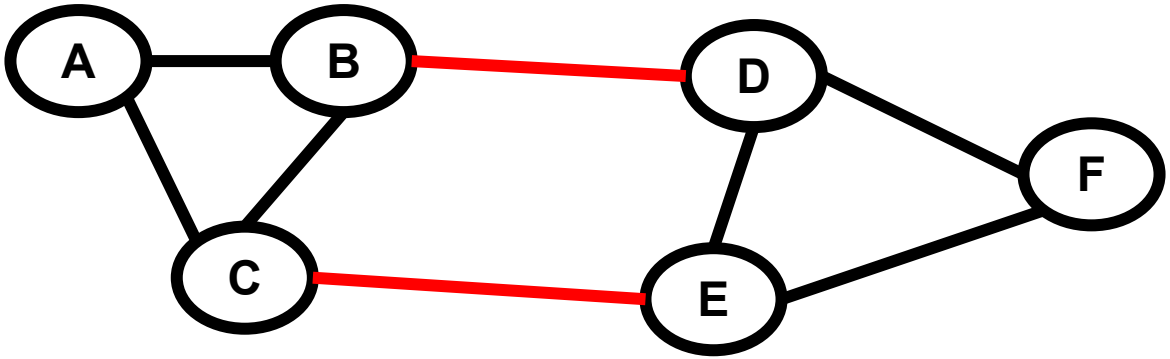
$$G = [e11 - (e11+e12)^2] + [e22 - (e21+e22)^2]$$



$$G = (2/8 - (4/8)^2) + (2/8 - (4/8)^2) = 0{,}00$$

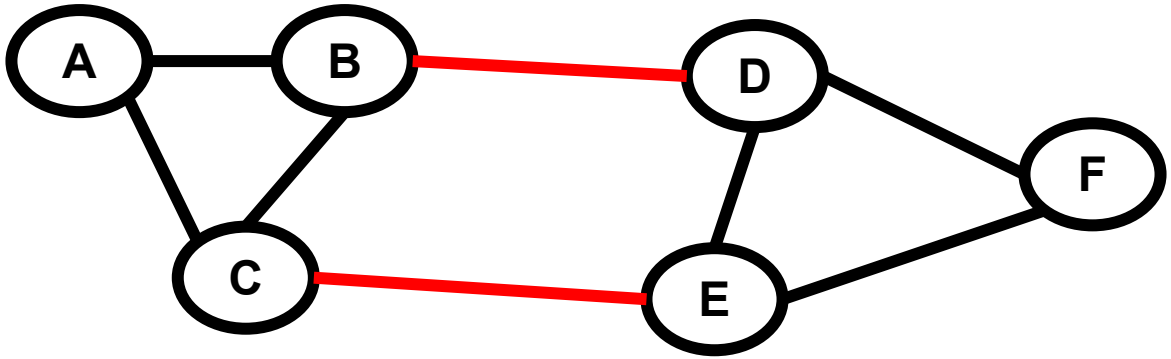
Universidade Federal do ABC

Maximizar modularidade



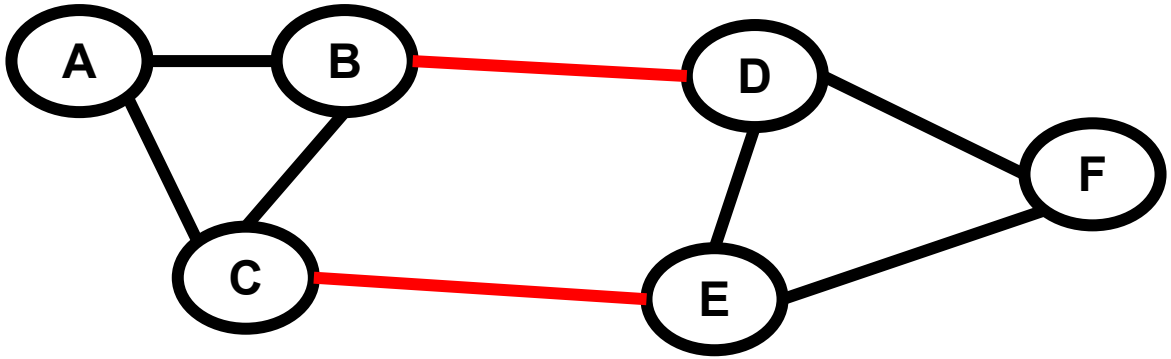
Maximizar modularidade

e11 = e22 = 3/8



Maximizar modularidade

e12 = e21 = 1/8

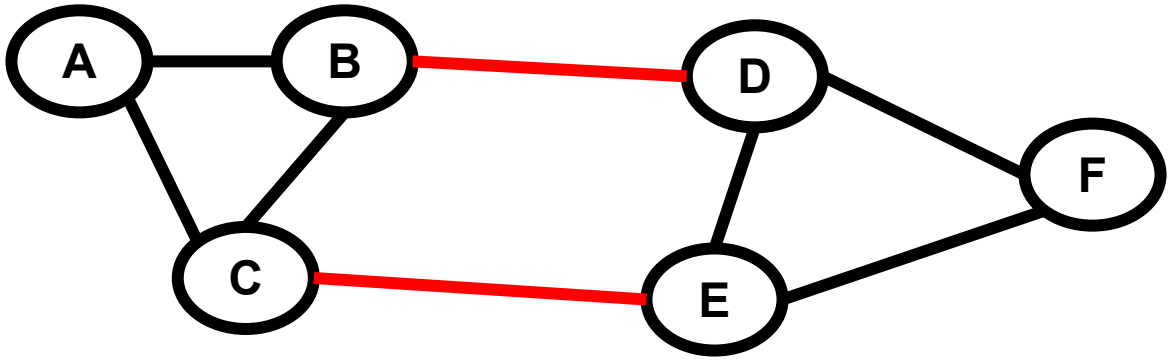


Maximizar modularidade

$$E = \begin{pmatrix} 3/8 & 1/8 \\ 1/8 & 3/8 \end{pmatrix}$$

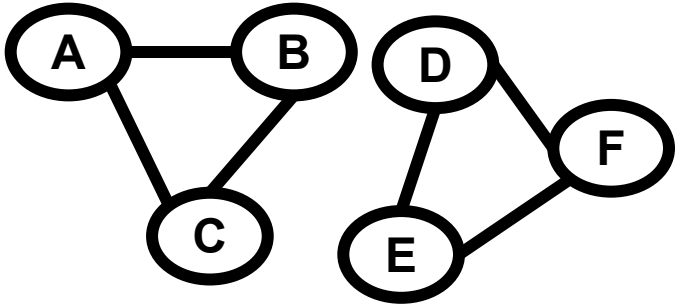
$$G = \left(e_{1,1} - (e_{1,1} + e_{1,2})^2\right) + \left(e_{2,2} - (e_{2,1} + e_{2,2})^2\right)$$

$$G = (3/8 - (4/8)^2) + (3/8 - (4/8)^2) = 0,25$$



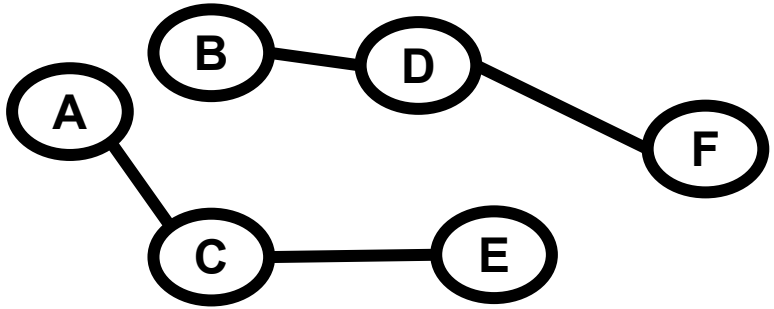
Universidade Federal do ABC

Modularidade



0,25

>



0,00

Algoritmo de Maximização de Modularidade

- Partição inicial: calcule G
- Para cada vizinho: calcule G
- Escolha vizinho de maior G
- Repita até não existir vizinho melhor

Algoritmo de Maximização de Modularidade

Amazon: “quem comprou X também comprou Y”.